Information Fusion Kalman Estimators with Different Local Models
Guangming Yan, Xiaojun Sun and Shi Guangfan

Abstract—For the multisensor linear discrete time-varying stochastic control systems with different local models, three distributed optimal fusion Kalman estimators weighted by matrices, diagonal matrices and scalars are presented in the linear minimum variance sense. They can handle the fused filtering, prediction and smoothing problems, and are applicable to the multisensor systems with colored measurement noise. They are locally optimal and are globally suboptimal. The accuracy of the fusers is higher than that of each local Kalman estimator. In order to compute the optimal weights, the new formula computing the cross-covariances among local smoothing errors is given. A Monte Carlo simulation example for the tracking system with colored measurement noises and 3 sensors shows their effectiveness.

Index Terms—Different local models, Kalman filter, Multisensor information fusion, Weighted fusion.

I. INTRODUCTION

In recent years, the multisensor information fusion Kalman filtering has been widely applied in many fields such as guidance, defense, robotics, integrated navigation, target tracking, GPS positioning, communication, signal processing and control etc [1]. For Kalman filtering-based fusion, two basic fusion methods are centralized and distributed fusion methods, depending on whether raw data are used directly for fusion or not [2]. The centralized fusion method can give the globally optimal state estimation by combining local measurement data. But its disadvantage is that it may require larger computational burden and higher data rates for communication. The distributed fusion method can give the globally optimal or suboptimal state estimation by combining the local state estimators. This method has considerable advantages that it can facilitate fault detection and isolation more conveniently, and can increase the input data rates significantly. There are two approaches for distributed fusion Kalman filtering, which are the information matrix approach [3,4] and the weighted covariance approach [5-9]. The distributed fusion Kalman filters with feedback and without feedback by the information matrix approach are equivalent to the centralized fusion Kalman filter, and give the globally optimal state estimation [4], but they all will require extensive calculations of local and global inverse covariances. The weighted covariance approach is a covariance-based weighted fusion approach with various weighted fusion rules. It can eliminate expensive computational requirements, but generally it gives a globally suboptimal state estimation with a slight loss of accuracy. So far, there are three optimal weighted fusion rules, which are the fusion rules weighted by matrices, diagonal matrices and scalars [7-9]. These fusion rules give the locally optimal estimators, which are globally suboptimal compared with the centralized fusion.

So far, the multisensor information fusion estimation is mainly focused on the filtering fusion, but the smoothing fusion is seldom reported [10], and the fusion estimation for multisensor systems with colored measurement noises is seldom reported [11].

Recently, using the modern time series analysis method, an information fusion steady-state Kalman estimator was presented in [9], but it is only applicable to the time-invariant systems with different local models. For the time-varying multisensor systems with the same local models and white measurement noises, Sun [10] presented a distributed fusion fixed-lag Kalman smoother with the components fusion weighted by scalars, which is equivalent to the fuser weighted by diagonal matrices [9], but it is not applicable to multisensor systems with colored measurement noises, and the fused Kalman smoothers weighted by matrices and scalars were not presented. A distributed fusion steady-state Kalman filter with matrix weights was presented for systems with colored measurement noises in [11], but the smoothing fusion problem was not solved. Sun [12] presented three weighted fusion Kalman smoothers for the time-varying systems with same local models.

In above references, the multisensor information fusion Kalman estimators for the time-varying systems with different local dynamic models have not been presented. For the multisensor information fusion time-varying systems with different local models, Sun [13,14] gave the information fusion Kalman smoothers weighted by matrices, diagonal matrices and
is the discrete time, $\Pi$ denotes the transpose, and $(*)_0$ is the Kronecker product. They are called filters, smoothers or predictors, respectively.

The objectives are to find the local optimal (linear minimum error covariance) Kalman filters, with mean $\hat{x}_i(t)$ and $\hat{x}_i(t)$, respectively.

Consider the multisensor linear discrete time-varying stochastic control system with different local models.

$$x_i(t+1) = \Phi_i(t)x_i(t) + B_i(t)u_i(t) + \Gamma_i(t)w_i(t)$$

$$z_i(t) = H_i(t)x(t) + \nu_i(t), i = 1, \cdots, L$$

where $x_i(t) \in R^{n_i}$, $\nu_i(t) \in R^{m_i}$ and $w_i(t) \in R^r$ are the state, measurement, input noise and measurement noise of the $i$th sensor subsystem, respectively. $x_i(t)$ is the common state of all sensor subsystems. $\Phi_i(t)$, $\Gamma_i(t)$, $H_i(t)$ and $C_i$ are common matrices with compatible dimensions.

**Assumption 1.** $w(t)$ and $\nu_i(t)$ are correlated white noise with zero mean and $E$ denotes the mathematical expectation, the superscript $T$ denotes the transpose, and $\delta_{ik}$ is the Kronecker delta function.

**Assumption 2.** The initial state $x_i(0)$ with mean $\mu_i$ and error covariance $P_{i0}$, $i = 1, \cdots, L$.

The objectives are to find the local optimal (linear minimum variance) Kalman estimators $\hat{x}_i(t)$. The state, measurement, input noise and measurement noise of the $i$th sensor subsystem, $i = 1, \cdots, L$, and to find the optimal fusion Kalman estimators $\tilde{x}_i(t)$. weighted by matrices, diagonal matrices and scalars based on the local Kalman estimators, respectively. For $N > 0$, $N > 0$ and $N < 0$, they are called filters, smoothers or predictors, respectively.

II. PROBLEM FORMULATION

The smoothing error covariances $(*)_0$ are the prediction and filtering gain matrices respectively.

The prediction error covariances $P_{i0}(t+1|t)$ satisfy the Riccati equation

$$P_i(t+1) = \Phi_i(t)P_i(t)[t-1] + K_i(t)S_i(t)Q_i(t)K_i(t)[t-1]^{-} \times [\Phi_i(t)P_i(t)[t-1] + K_i(t)S_i(t)]^{-} + G_i(t)Q_i(t)G_i(t)^{-}$$

with initial value $P_i(0) = 0$. $i = 1, \cdots, L$, $t \neq k$.

**Proof.** From [15], we have the prediction error equation

$$\hat{x}_i(t+1) = \Phi_i(t)\hat{x}_i(t) + \nu_i(t)$$

where $w_i(t)$ is independent of $\nu_i(t)$ and $\hat{x}_i(t)$. Using (14), we obtain (12).

**Lemma 2.** For the multisensor time-varying system (1)-(3) with the assumptions 1 and 2, the $i$th sensor subsystem has the local optimal Kalman filter $\hat{x}_i(t|t+N)$, $N = 0$ and smoother $\tilde{x}_i(t|t+N)$, $N > 0$.

$$\hat{x}_i(t|t+N) = \hat{x}_i(t|t-1) + \sum_{k=0}^{N} K_i(t|t+k)\epsilon_i(t+k)$$

$$\tilde{x}_i(t|t+N) = \hat{x}_i(t|t-1) + \sum_{k=0}^{N} K_i(t|t+k)\epsilon_i(t+k)$$

where $i = 1, \cdots, L$, $t \neq k$, and $\epsilon_i(t)$ are the local measurement noise.

**Theorem 1.** For the multisensor time-varying system (1)-(3) with the assumptions 1 and 2, the cross-covariances among local prediction errors are given as

$$\sum_{k=0}^{N} K_i(t|t+k)\epsilon_i(t+k)$$

**Proof.** The proof is omitted.
Proof. The proof is given in [15], which is omitted.

**Theorem 2.** For the multisensor time-varying system with the assumptions 1 and 2, the cross-covariances among local smoothing errors are given as

$$\mathbb{E}\left[\mathbf{y}_i(t)\right] = \mathbb{E}\left[\mathbf{y}_i(t)\right] = I_n$$

where we define

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (19), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (20), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (21), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (22), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (23), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (24), we have the following equation

$$\mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T] = I_n$$

where

$$\mathbf{y}_i(t) = \mathbb{E}[\mathbf{y}_i(t)\mathbf{y}_i(t)^T].$$

**Proof.** From (25), we obtain (23). From (26), we obtain (24). From (27), we obtain (28). From (28), we obtain (29). From (29), we obtain (30). From (30), we obtain (31). From (31), we obtain (32). From (32), we obtain (33). From (33), we obtain (34). From (34), we obtain (35). From (35), we obtain (36). From (36), we obtain (37). From (37), we obtain (38). From (38), we obtain (39). From (39), we obtain (40). From (40), we obtain Theorem 3.
IV. SIMULATION EXAMPLE

Consider the 3-sensor discrete time-varying tracking system with colored measurement noises

\[ x(t+1) = F(x(t) + \Gamma w(t)) \]  
\[ y_i(t) = Hx(t) + \eta_i(t) + v_i(t), \quad i = 1, 2, 3 \]  
(41)

where \( q^{-1} \) is the backward shift operator, \( \eta(t) = \eta(t-1) \), \( T_0 \) is the sampled period, \( x(t) = [x_i(t), x_i(t)]^T \) is the state. \( x_i(t) \), \( x_i(t) \) are the position and velocity of target at time \( t \), respectively. \( w(t) \), \( v(t) \) and \( \xi_i(t) \) are independent white noises with zero means and variance matrices \( \sigma^2_i \), \( \sigma^2_j \) and \( \sigma^2_k \), respectively. The problem is to compare the accuracy of the local and fused Kalman smoothers of the state \( x(t) \).

Transform (43) into the state-space model as

\[ \alpha_i(t+1) = A_i(t)\alpha_i(t) + R_k(t) \]  
(45)

\[ \eta_i(t) = H_i \alpha_i(t) \]  
(46)

Defining \( A_i(t) = \begin{bmatrix} -a_{i1}(t) & 1 \\ -a_{i2}(t) & 0 \end{bmatrix} \), \( R = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), \( H_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \).

Combining (41) and (45), we have the system with different local models

\[ x_i(t+1) = \Phi_i(t)x_i(t) + \Gamma w_i(t) \]  
(47)

\[ y_i(t) = H_i x_i(t) + v_i(t), \quad i = 1, 2, 3 \]  
(48)

\[ x(t) = x_i(t) = C_i x_i(t) \]  
(49)

with definitions

\[ \Phi_i(t) = \begin{bmatrix} \Phi & 0 \\ 0 & A_i(t) \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad H_i = \begin{bmatrix} H_i \\ H_i \end{bmatrix}, \]

so the problem is converted to that of the accuracy comparison of local and fused Kalman smoothers for the common state \( x_i(t) = x(t) \) for the system (47)-(49).

The simulation results are shown in Fig. 1, Fig. 2 and Table I. In Fig. 1 and Table I, we can see that the accuracy of fused smoothers is higher than that of each local smoother, and the theoretical accuracy relation (40) holds. But the accuracy distinction of the three weighted fusers is not obvious, because Fig. 1 shows that the curves of \( \text{tr} P_{60}^\theta (t+1|t+2) \) are as indistinguishable. 300 Monte Carlo runs are carried out and the means square error (MSE) curves is shown in the Fig. 2, where the MSE value at \( t \) is defined as

\[ \text{MSE}_i(t) = \frac{1}{m} \sum_{j=1}^{m} (x_i^j(t+1|t+2) - \hat{x}_i^j(t|t+2))^T (x_i^j(t+1|t+2) - \hat{x}_i^j(t|t+2)) \]  
(68)

where \( x_i^j(t+1|t+2) = x_i(t+1|t+2) = x_i(t+1|t+2) \), \( i = 0, 1, 2, 3 \), \( t = 1, \ldots, 200 \), \( x_i^j(t+1|t+2) \) is the \( j \)-th sample of the stochastic process \( x_i(t+1|t+2) \), \( j = 1, \ldots, m \), \( m = 300 \) is the sampled number. From Fig. 2, we also see that the accuracy of the fusers is higher than that of each local smoother, the accuracy distinction of the three weighted fusers is not obvious, because their MSE curves are as indistinguishable.
TABLE I
COMPARISON OF LOCAL TRACES $\text{tr}(P_t | t + 2)$ AND FUSED TRACES $\text{tr}(P^\theta_t | t + 2), \theta = s, d, m$

<table>
<thead>
<tr>
<th></th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{tr}(P_{ts}^s</td>
<td>t + 2)$</td>
<td>1.255432</td>
<td>1.058630</td>
<td>1.255431</td>
</tr>
<tr>
<td>$\text{tr}(P_{ts}^d</td>
<td>t + 2)$</td>
<td>1.931443</td>
<td>1.672700</td>
<td>1.931445</td>
</tr>
<tr>
<td>$\text{tr}(P_{ts}^m</td>
<td>t + 2)$</td>
<td>2.782587</td>
<td>2.451384</td>
<td>2.782579</td>
</tr>
<tr>
<td>$\text{tr}(P_{ts}^s</td>
<td>t + 2)$</td>
<td>0.846548</td>
<td>0.733014</td>
<td>0.846549</td>
</tr>
<tr>
<td>$\text{tr}(P_{ts}^d</td>
<td>t + 2)$</td>
<td>0.846512</td>
<td>0.732951</td>
<td>0.846513</td>
</tr>
<tr>
<td>$\text{tr}(P_{ts}^m</td>
<td>t + 2)$</td>
<td>0.846223</td>
<td>0.732148</td>
<td>0.846224</td>
</tr>
</tbody>
</table>

V. CONCLUSION

For the linear discrete time-varying stochastic control systems with different local models, using the Kalman filtering method, three optimal information fusion Kalman estimators weighted by matrices, diagonal matrices and scalars have been presented in the linear minimum variance sense. They are locally optimal, and are globally suboptimal. In order to compute the optimal weights, the new formula of computing the cross-covariances among local estimation errors was presented. A Monte Carlo simulation example shows the accuracy distinction of three Kalman fusers is not obvious, so that the Kalman fuser with scalar weights or diagonal matrices is more suitable for real time applications. The proposed results can handle the fused estimation for multisensor systems with colored measurement noise.

REFERENCES